# T-Wise Test Generation and Prioritization For Large Software Product Line: Technical Report 

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#### Abstract

This document presents the distance and fitness functions used in the paper entitled "Bypassing the Combinatorial Explosion: Using Similarity to Generate and Prioritize T-wise Test Suites for Large Software Product Lines". In particular, it is shown how these functions are useful in terms of $t$-wise coverage.


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## 1 Representation of the products of a feature model

One product or one configuration is represented as a set of $n$ features of a feature model as $P=\left\{ \pm f_{1}, \ldots, \pm f_{n}\right\}$, where $+f_{i}$ indicates a feature which is selected by this product, and $-f_{i}$ an unselected one. Table 1 illustrates an example of three products and four features.

For instance, product $P_{1}=\left\{+f_{1},+f_{2},+f_{3},-f_{4}\right\}$ supports all the features except $f_{4}$.

|  |  | Features |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $f_{1}$ | $f_{2}$ | $f_{3}$ | $f_{4}$ |  |
| Products | $P_{1}$ | $\times$ | $\times$ | $\times$ |  |  |
|  | $P_{2}$ | $\times$ | $\times$ |  | $\times$ |  |
|  | $P_{3}$ | $\times$ |  | $\times$ |  |  |

Table 1: Example of Three Products For an FM of Four Features

## 2 A distance between two prodcuts

We use the Jaccard distance [1] $d$ between two products of $n$ features:

$$
d: \begin{aligned}
& P \times P \longrightarrow[0,1] \\
&\left(P_{i}, P_{j}\right) \longmapsto P_{i} \cap P_{j} \\
& \# P_{i} \cup P_{j}
\end{aligned},
$$

where $P_{i}, P_{j} \in P$, where $n$ denotes the number of features of the feature model, where $\# A$ denotes the cardinality of the set $A$ and with $\# P_{i} \cup P_{j}>0$.

For instance, with reference to Table 1, $n=4$ and $P_{1}=\left\{+f_{1},+f_{2},+f_{3},-f_{4}\right\}, P_{2}=\left\{+f_{1},+f_{2},-f_{3},+f_{4}\right\}$ and $P_{3}=\left\{+f_{1},-f_{2},+f_{3},-f_{4}\right\}$. Thus:

- $d\left(P_{1}, P_{2}\right)=1-\frac{\#\left\{+f_{1},+f_{2}\right\}}{\#\left\{+f_{1},+f_{2},+f_{3},-f_{3},+f_{4},-f_{4}\right\}}=1-\frac{2}{6} \approx 0.67$,
- $d\left(P_{1}, P_{3}\right)=0.4$,
- $d\left(P_{2}, P_{3}\right)=\approx 0.86$.

In this example, $P_{1}$ and $P_{3}$ are the most similar products (i.e. they share the lowest distance), whereas $P_{2}$ and $P_{3}$ are the most dissimilar ones.

Besides, the distance $d$ is a normalization between 0 and 1 of the following distance $d^{\prime}$ :

$$
\begin{array}{rlll}
d^{\prime}: & P \times P & \longrightarrow & {\left[0, \# P_{i} \cup P_{j}\right]} \\
\left(P_{i}, P_{j}\right) & \longmapsto & \# P_{i} \cup P_{j}-\# P_{i} \cap P_{j} .
\end{array}
$$

In our context, the exact distances values are not important. What is relevant for us is the order between two distances, meaning that if we have $N$ products and $k$ Jaccard distances, we have $d^{1}\left(P_{1}, P_{2}\right)<$ $\ldots<d^{k}\left(P_{N-1}, P_{N}\right) \Leftrightarrow d^{\prime 1}\left(P_{1}, P_{2}\right)<\ldots<d^{\prime k}\left(P_{N-1}, P_{N}\right)$ because the normalization does not affect the ordering between the distances. In a similar way, $d^{1}\left(P_{1}, P_{2}\right)>\ldots>d^{k}\left(P_{N-1}, P_{N}\right) \Leftrightarrow d^{11}\left(P_{1}, P_{2}\right)>\ldots>$ $d^{\prime k}\left(P_{N-1}, P_{N}\right)$. In the following, we'll use $d^{\prime}$ to simplify the demonstration.

### 2.1 Evaluating the $t$-wise coverage of two products

The $t$-wise coverage expresses the ability of the products to cover the $t$-sets of features of the feature model. A $t$-set is a valid combination of $t$ (selected or unselected) features of the feature model. This coverage can be evaluated as the number of $t$-set covered by the products.

Each feature model of $n$ features has at most $\binom{2 n}{t}$ valid $t$-sets. Each product $P$ covers exactly $C(P, n, t)=$ $C(P, \# P, t)=\binom{n}{t} t$-sets. As a result, the $t$-wise coverage of two products $P_{i}$ and $P_{j}$ is defined by the number of $t$-sets covered by these two products minus the common $t$-sets covered by these products:

$$
\begin{aligned}
\operatorname{Coverage}\left(P_{i}, P_{j}\right) & =C\left(P_{i}, \# P_{i}, t\right)+C\left(P_{j}, \# P_{j}, t\right)-C\left(P_{i} \cap P_{j}, \# P_{i} \cap P_{j}, t\right) \\
& =C\left(P_{i}, \# P_{i}, t\right)+C\left(P_{j}, \# P_{j}, t\right)-C(P_{i} \cap P_{j}, \underbrace{\# P_{i} \cup P_{j}-d^{\prime}\left(P_{i}, P_{j}\right)}_{x}, t)
\end{aligned}
$$

Proposition 1. The distance $d^{\prime}$ (and thus d) is a good indicator of the $t$-wise coverage between two products, i.e. Coverage $\left(P_{i}, P_{j}\right)$ increases with $d^{\prime}\left(P_{i}, P_{j}\right)$.

Proof. We have, in particular:

- When $d^{\prime}\left(P_{i}, P_{j}\right)=0$, then $x=n$ and $\# P_{i} \cap P_{j}=n$. Thus, $P_{i}=P_{j}$ and the coverage is at its minimum value: $\operatorname{Coverage}\left(P_{i}, P_{j}\right)=C\left(P_{i}, n, t\right)=C\left(P_{j}, n, t\right)=\binom{n}{t} t$-sets.
- When $d^{\prime}\left(P_{i}, P_{j}\right)=n$, then $x=n$ and $\# P_{i} \cap P_{j}=0$. Thus, the coverage is at its maximum value: $\operatorname{Coverage}\left(P_{i}, P_{j}\right)=C\left(P_{i}, n, t\right)+C\left(P_{j}, n, t\right)=2\binom{n}{t} t$-sets.

Thus:

- Coverage $\left(P_{i}, P_{j}\right) \geq \operatorname{Coverage}\left(P_{i}^{\prime}, P_{j}^{\prime}\right) \Leftrightarrow d^{\prime}\left(P_{i}, P_{j}\right) \geq d^{\prime}\left(P_{i}^{\prime}, P_{j}^{\prime}\right)$.
- Coverage $\left(P_{i}, P_{j}\right) \leq \operatorname{Coverage}\left(P_{i}^{\prime}, P_{j}^{\prime}\right) \Leftrightarrow d^{\prime}\left(P_{i}, P_{j}\right) \leq d^{\prime}\left(P_{i}^{\prime}, P_{j}^{\prime}\right)$.

As a result, the above-mentioned distance $d^{\prime}$ (and thus $d$ ) is relevant to evaluate the coverage of two products without computing the $t$-sets.

## 3 A fitness function for $N$ products

We define a fitness function $f$ based on $d$ to evaluate the quality of $N$ products $P_{1}, \ldots, P_{N}$ :

$$
\begin{aligned}
P^{N} & \longrightarrow \mathbb{R}_{+} \\
\left(P_{1}, \ldots, P_{N}\right) & \longmapsto \sum_{j>i \geq 1}^{N} d\left(P_{i}, P_{j}\right) .
\end{aligned}
$$

### 3.1 Comparing two $N$-uplets of products in terms of $t$-wise coverage

For a given and fixed value of $N$, this fitness function allows to choose between two $N$-uplets products $S_{1}=\left(P_{1}, \ldots, P_{N}\right)$ and $S_{2}=\left(P_{1}^{\prime}, \ldots, P_{N}^{\prime}\right)$ which set potentially has the highest $t$-sets coverage. To this end, we evaluate $f\left(S_{1}\right)$ and $f\left(S_{2}\right)$ and we take the set where $f$ is maximum.

Proposition 2. The fitness function $f$ is a good indicator of the $t$-wise coverage between $N \geq 2$ products, i.e. Coverage $\left(P_{1}, \ldots P_{N}\right)$ generally increases with $f\left(P_{1}, \ldots, P_{N}\right)$.

Proof. The fitness function $d$ corresponds to the sum of the distances pairwise between the products:

$$
f\left(P_{1}, \ldots, P_{N}\right)=d\left(P_{1}, P_{2}\right)+d\left(P_{1}, P_{3}\right)+\ldots+d\left(P_{N-1}, P_{N}\right) .
$$

In other words, $f$ corresponds to the evaluation of $d$ between each couple of products. Since it's a sum and given the properties of $d$, it is direct that the value of $f$ will be maximum when all the terms of the sum are maximum, i.e. when the distance $d$ among any two products is maximum.

## References

[1] Paul Jaccard. Étude comparative de la distribution florale dans une portion des alpes et des jura. Bulletin del la Société Vaudoise des Sciences Naturelles, 37:547-579, 1901.

