# T-Wise Test Generation and Prioritization For Large Software Product Line: Technical Report

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#### Abstract

This document presents the distance and fitness functions used in the paper entitled "Bypassing the Combinatorial Explosion: Using Similarity to Generate and Prioritize T-wise Test Suites for Large Software Product Lines". In particular, it is shown how these functions are useful in terms of *t*-wise coverage.

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## 1 Representation of the products of a feature model

One product or one configuration is represented as a set of n features of a feature model as  $P = \{\pm f_1, ..., \pm f_n\}$ , where  $+f_i$  indicates a feature which is selected by this product, and  $-f_i$  an unselected one. Table 1 illustrates an example of three products and four features.

For instance, product  $P_1 = \{+f_1, +f_2, +f_3, -f_4\}$  supports all the features except  $f_4$ .

	Features				
		$f_1$	$f_2$	$f_3$	$f_4$
Products	$P_1$	×	×	X	
	$P_2$	×	×		Х
	$P_3$	×		×	

Table 1: Example of Three Products For an FM of Four Features

#### 2 A distance between two prodcuts

We use the Jaccard distance [1] d between two products of n features:

$$d: \begin{array}{ccc} P \times P & \longrightarrow & [0,1] \\ d: & (P_i, P_j) & \longmapsto & 1 - \frac{\# P_i \cap P_j}{\# P_i \cup P_i}, \end{array}$$

where  $P_i, P_j \in P$ , where *n* denotes the number of features of the feature model, where #A denotes the cardinality of the set *A* and with  $#P_i \cup P_j > 0$ .

For instance, with reference to Table 1, n = 4 and  $P_1 = \{+f_1, +f_2, +f_3, -f_4\}$ ,  $P_2 = \{+f_1, +f_2, -f_3, +f_4\}$  and  $P_3 = \{+f_1, -f_2, +f_3, -f_4\}$ . Thus:

- $d(P_1, P_2) = 1 \frac{\#\{+f_1, +f_2\}}{\#\{+f_1, +f_2, +f_3, -f_3, +f_4, -f_4\}} = 1 \frac{2}{6} \approx 0.67,$
- $d(P_1, P_3) = 0.4,$

•  $d(P_2, P_3) = \approx 0.86.$ 

In this example,  $P_1$  and  $P_3$  are the most similar products (i.e. they share the lowest distance), whereas  $P_2$  and  $P_3$  are the most dissimilar ones.

Besides, the distance d is a normalization between 0 and 1 of the following distance d':

$$d': \begin{array}{ccc} P \times P & \longrightarrow & [0, \#P_i \cup P_j] \\ (P_i, P_j) & \longmapsto & \#P_i \cup P_j - \#P_i \cap P_j. \end{array}$$

In our context, the exact distances values are not important. What is relevant for us is the order between two distances, meaning that if we have N products and k Jaccard distances, we have  $d^1(P_1, P_2) < \ldots < d^k(P_{N-1}, P_N) \Leftrightarrow d'^1(P_1, P_2) < \ldots < d'^k(P_{N-1}, P_N)$  because the normalization does not affect the ordering between the distances. In a similar way,  $d^1(P_1, P_2) > \ldots > d^k(P_{N-1}, P_N) \Leftrightarrow d'^1(P_1, P_2) > \ldots > d'^k(P_{N-1}, P_N)$ . In the following, we'll use d' to simplify the demonstration.

#### 2.1 Evaluating the *t*-wise coverage of two products

The t-wise coverage expresses the ability of the products to cover the t-sets of features of the feature model. A t-set is a valid combination of t (selected or unselected) features of the feature model. This coverage can be evaluated as the number of t-set covered by the products.

Each feature model of *n* features has at most  $\binom{2n}{t}$  valid *t*-sets. Each product *P* covers exactly  $C(P, n, t) = C(P, \#P, t) = \binom{n}{t}$  *t*-sets. As a result, the *t*-wise coverage of two products  $P_i$  and  $P_j$  is defined by the number of *t*-sets covered by these two products minus the common *t*-sets covered by these products:

$$Coverage(P_i, P_j) = C(P_i, \#P_i, t) + C(P_j, \#P_j, t) - C(P_i \cap P_j, \#P_i \cap P_j, t) = C(P_i, \#P_i, t) + C(P_j, \#P_j, t) - C(P_i \cap P_j, \underbrace{\#P_i \cup P_j - d'(P_i, P_j)}_{T_i}, t).$$

**Proposition 1.** The distance d' (and thus d) is a good indicator of the t-wise coverage between two products, *i.e.* Coverage $(P_i, P_j)$  increases with  $d'(P_i, P_j)$ .

*Proof.* We have, in particular:

- When  $d'(P_i, P_j) = 0$ , then x = n and  $\#P_i \cap P_j = n$ . Thus,  $P_i = P_j$  and the coverage is at its minimum value:  $Coverage(P_i, P_j) = C(P_i, n, t) = C(P_j, n, t) = \binom{n}{t}$  t-sets.
- When  $d'(P_i, P_j) = n$ , then x = n and  $\#P_i \cap P_j = 0$ . Thus, the coverage is at its maximum value:  $Coverage(P_i, P_j) = C(P_i, n, t) + C(P_j, n, t) = 2\binom{n}{t} t$ -sets.

Thus:

- $Coverage(P_i, P_j) \ge Coverage(P'_i, P'_j) \Leftrightarrow d'(P_i, P_j) \ge d'(P'_i, P'_j).$
- $Coverage(P_i, P_j) \leq Coverage(P'_i, P'_j) \Leftrightarrow d'(P_i, P_j) \leq d'(P'_i, P'_j).$

As a result, the above-mentioned distance d' (and thus d) is relevant to evaluate the coverage of two products without computing the *t*-sets.

## **3** A fitness function for N products

We define a fitness function f based on d to evaluate the quality of N products  $P_1, ..., P_N$ :

$$f: \begin{array}{ccc} P^N & \longrightarrow & \mathbb{R}_+ \\ (P_1, ..., P_N) & \longmapsto & \sum_{j>i\geq 1}^N d(P_i, P_j). \end{array}$$

#### 3.1 Comparing two *N*-uplets of products in terms of *t*-wise coverage

For a given and fixed value of N, this fitness function allows to choose between two N-uplets products  $S_1 = (P_1, ..., P_N)$  and  $S_2 = (P'_1, ..., P'_N)$  which set potentially has the highest *t*-sets coverage. To this end, we evaluate  $f(S_1)$  and  $f(S_2)$  and we take the set where f is maximum.

**Proposition 2.** The fitness function f is a good indicator of the t-wise coverage between  $N \ge 2$  products, *i.e.* Coverage $(P_1, ..., P_N)$  generally increases with  $f(P_1, ..., P_N)$ .

*Proof.* The fitness function d corresponds to the sum of the distances pairwise between the products:

 $f(P_1, \dots, P_N) = d(P_1, P_2) + d(P_1, P_3) + \dots + d(P_{N-1}, P_N).$ 

In other words, f corresponds to the evaluation of d between each couple of products. Since it's a sum and given the properties of d, it is direct that the value of f will be maximum when all the terms of the sum are maximum, i.e. when the distance d among any two products is maximum.

## References

 Paul Jaccard. Étude comparative de la distribution florale dans une portion des alpes et des jura. Bulletin del la Société Vaudoise des Sciences Naturelles, 37:547–579, 1901.